

# New Bubble Point Pressure Correlation for Middle East Crude Oils

Sayed Gomaa<sup>1,2</sup>, PhD

Mining and Petroleum Engineering Department, Faculty of Engineering, Al-Azhar University, Cairo, Egypt<sup>1</sup>

Petroleum Engineering and Gas Technology Department, Faculty of Engineering, British University in Egypt<sup>2</sup>

**Abstract:** In the absence of the experimentally measured bubble-point pressure, it is necessary for the engineer to make an estimate of this crude oil property from the readily available measured producing parameters. Several authors have proposed graphical and mathematical correlations for determining the bubble point pressure. They presented their correlations based on the assumption that the bubble-point pressure is a strong function of solution gas-oil ratio, gas gravity, oil gravity, and temperature. This paper presents a comparison study among 23 different correlations used to calculate the bubble point pressure for 441 sample of Middle East crude oil. A new correlation was developed for calculating the bubble point with correlation coefficient of 0.98.

**Keywords:** Bubble point pressure, Gas specific gravity, Solution GOR, Reservoir temperature, API.

## INTRODUCTION

Bubble point pressure is very important for reservoir and production engineers. If you want to calculate the original oil in place or predict the future production performance using material balance equation or if you plan for waterflood project for example, you must know the bubble point pressure.

Many authors have proposed different correlations for calculating the bubble point pressure. The engineers use these correlations in the absence of PVT data.

Table 1 depicts a summary of 23 published correlation for calculating the bubble point pressure including the data range for each correlation.

**Table 1. Summary of published bubble point correlations**

Author	Correlation	Reference
Standing (1947)	$p_b = 18.2 \left[ (R_s/\gamma_g)^{0.83} 10^x - 1.4 \right]$ $x = 0.00091(T - 460) - 0.0125API$	1
Lasater (1958)	$M_o = (26.297 - 0.2706API)^2$ $Y_g = (R_s/379.4)/(R_s/379.4 + 350\gamma_o/M_o)$ $p_f = 0.046 + 2.273 Y_g + 7.522Y_g^3$ $p_b = p_f(T/\gamma_g)$	2
Vazquez (1980)	$p_b = [a_1 R_s/\gamma_{gs} 10^x]^{a_2}$ $x = a_3 API / T$ $API \leq 30 : a_1 = 27.624 : a_2 = 0.914328 : a_3 = 11.712$ $API > 30 : a_1 = 56.18 : a_2 = 0.84246 : a_3 = 10.393$	3
Glaso (1980)	$\log(p_b) = 1.7669 + 1.7447 \log(x) - 0.30218 [\log(x)]^2$ $x = (R_s/\gamma_g)^{0.816} T^{0.172} API^{-0.989}$	4
Al-Marhoun (1988)	$p_b = a_1 \gamma_g^{a_2} API^{a_3} R_s^{a_4} T^{a_5}$ $a_1 = 0.00538088 \quad a_2 = -1.87784 \quad a_3 = -1.0305$ $a_4 = 0.57743 \quad a_5 = 0.6641$	5
Dokla and Osman (1992)	$p_b = a_1 \gamma_g^{a_2} \gamma_o^{a_3} R_s^{a_4} T^{a_5}$ $a_1 = 8363.86 \quad a_2 = -1.01049 \quad a_3 = 0.107991$ $a_4 = 0.724047 \quad a_5 = 0.952584$	6
Macary and El-Batanony (1992)	$p_b = 204.257 x (R_s^{0.5} - 4.7927)$ $x = \text{Exp}(0.00077T - 0.0097API - 0.4003\gamma_g)$	7

Omar and Todd (1993)	$p_b = 18.2 \left[ \left( \frac{R_s}{\gamma_g} \right)^x 10^{(0.00091T - 0.0125API)} - 1.4 \right]$ $x = 1.4256 - 0.2608B_o - 0.4596\gamma_g + 0.04481B_o^2 + 0.236\gamma_g^2 - 0.1077/(\gamma_g B_o)$	8																														
Petrosky and Farshad (1993)	$p_b = \frac{112.727R_s^{0.577421}}{\gamma_g^{0.8439} \times 10^X} - 1391.051$ $x = 0.0007916API^{1.541} - 0.00004561(T - 460)^{1.3911}$	9																														
Kartaotmodjo and Schmidt (1994)	$p_b = \left( \frac{R_s}{(a_1\gamma_g^{a_2} 10^{a_3API/T})} \right)^{a_4}$ <p><math>API \leq 30 : a_1 = 0.05958 : a_2 = 0.7972 : a_3 = 13.1405 : a_4 = 0.9986</math></p> <p><math>API &gt; 30 : a_1 = 0.0315 : a_2 = 0.7587 : a_3 = 11.28 : a_4 = 0.9145</math></p>	10																														
Ghetto (1994)	$p_b = 21.7429(R_s/\gamma_g)^{0.7646} 10^{0.00119T - 0.0101API}$	11																														
Farshad (1996)	$\log(p_b) = 0.3058 + 1.9013 \log(x) - 0.26(\log(x))^2$ $x = \gamma_g^{-1.378} R_s^{1.053} 10^{(0.00069T - 0.0208API)}$	12																														
Almehaideb (1997)	$p_b = -620.592 + \frac{6.23087R_s\gamma_o}{\gamma_g B_o^{1.38559}} + 2.89868T$	13																														
Hanafy (1997)	$p_b = 3.205R_{si} + 157.27$	14																														
Khairy (1998)	$p_b = a_1 \gamma_g^{a_2} \gamma_o^{a_3} R_s^{a_4} T^{a_5}$ <p><math>a_1 = 49.3647 \quad a_2 = -1.4676 \quad a_3 = 3.1437</math></p> <p><math>a_4 = 0.715082 \quad a_5 = 1.32657</math></p>	15																														
Al-Shammasi (1999)	$p_b = \gamma_o^{5.527215} \text{Exp}(1.841408\gamma_g\gamma_o) (\gamma_g R_s T)^{0.783716}$	16																														
Velarde and McCain (1999)	$p_b = \frac{1091.47R_s^{0.081465}}{\gamma_g^{0.161488} \times 10^X} - 0.740152$ $x = 8.2 \times 10^{-6} API^{2.176124} - 0.013098(T - 460)^{0.282372}$	17																														
Dindoruk and Christman (2001)	$p_b = a_8 \left[ \frac{R_s^{a_9}}{\gamma_g^{a_{10}}} \times 10^X + a_{11} \right]$ $x = \frac{a_1 T^{a_2} + a_3 API^{a_4}}{a_5 + \frac{2R_s^{a_6}}{\gamma_g^{a_7}}}$ <p><math>a_1 = 0.142828 \times 10^{-10} \quad a_2 = 2.844591797</math></p> <p><math>a_3 = -6.74896 \times 10^{-5} \quad a_4 = 1.225226436</math></p> <p><math>a_5 = 0.33383304 \quad a_6 = -0.272945957</math></p> <p><math>a_7 = 0.084226069 \quad a_8 = 1.869979257</math></p> <p><math>a_9 = 1.221486524 \quad a_{10} = -1.370508349</math></p> <p><math>a_{11} = 0.011688308</math></p>	18																														
McCain (2003)	$\ln(p_b) = 7.475 + 0.713z + 0.0075z^2$ $z = \sum_{n=1}^4 z_n : z_n = C_{0n} + C_{1n} VAR_n + C_{2n} VAR_n^2 + C_{3n} VAR_n^3$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>n</th> <th>VAR</th> <th>C<sub>0</sub></th> <th>C<sub>1</sub></th> <th>C<sub>2</sub></th> <th>C<sub>3</sub></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>ln(R<sub>s</sub>)</td> <td>-5.48</td> <td>-0.0378</td> <td>0.281</td> <td>-0.0206</td> </tr> <tr> <td>2</td> <td>API</td> <td>1.27</td> <td>-0.0449</td> <td>4.36 × 10<sup>-4</sup></td> <td>-4.76 × 10<sup>-6</sup></td> </tr> <tr> <td>3</td> <td>γ<sub>g</sub></td> <td>4.51</td> <td>-10.84</td> <td>8.39</td> <td>-2.34</td> </tr> <tr> <td>4</td> <td>T</td> <td>-0.7835</td> <td>6.23 × 10<sup>-3</sup></td> <td>-1.22 × 10<sup>-5</sup></td> <td>1.03 × 10<sup>-8</sup></td> </tr> </tbody> </table>	n	VAR	C <sub>0</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	1	ln(R <sub>s</sub> )	-5.48	-0.0378	0.281	-0.0206	2	API	1.27	-0.0449	4.36 × 10 <sup>-4</sup>	-4.76 × 10 <sup>-6</sup>	3	γ <sub>g</sub>	4.51	-10.84	8.39	-2.34	4	T	-0.7835	6.23 × 10 <sup>-3</sup>	-1.22 × 10 <sup>-5</sup>	1.03 × 10 <sup>-8</sup>	19
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Mehran (2006)	$p_b = a_1 \gamma_g^{a_2} \gamma_o^{a_3} R_s^{a_4} T^{a_5}$ $a_1 = 3.146 \quad a_2 = -1.3114 \quad a_3 = 3.3925$ $a_4 = 0.8035 \quad a_5 = 0.3466$	20
Hemmati and Kharrat (2007)	$p_b = 10.4566 \left[ \left( \frac{R_s}{\gamma_g} \right)^x 10^{0.0008T - 0.0098\gamma_o} - 8.6817 \right]$ $x = a_1 + a_2 B_o + a_3 \gamma_g + a_4 B_o^2 + a_5 \gamma_g^2 + a_6 / (\gamma_g B_o)$ $a_1 = 1.5897 \quad a_2 = 0.2735 \quad a_3 = 0.4429$ $a_4 = 0.04692 \quad a_5 = 0.144 \quad a_6 = 0.1596$	21
Ikiensikimama and Ogboja (2009)	$Q = (R_s/a_1) / (R_s/a_1 + a_2 \gamma_o / M_o)$ $S = (a_3 - a_4 API)^{a_5}$ $p_b^* = a_6 + a_7 Q + a_8 Q^{a_9}$ $p_b = p_b^* (T + a_{10}) / \gamma_g$ $a_1 = 336.0064009 \quad a_2 = 6.7063984 \quad a_3 = 47.57094772$ $a_4 = 0.677706662 \quad a_5 = 1.530935619$ $a_6 = 0.243181338 \quad a_7 = -2.316548789 \quad a_8 = 10.60657909$ $a_9 = 1.518030465 \quad a_{10} = 635.4152349$	22
Moradi (2010)	$p_b^* = 1.10382622244782 \log_{10}(API) \times API^{6.20868199092533}$ $\times \text{Exp}(-1.84068688374902 \gamma_g^2)$ $\times (TR_s \gamma_g)^{0.688750576134232}$ $p_b = -65.853149 p_b^* + 0.00040668902 p_b^{*2}$ $- 0.00000015472455 p_b^{*3}$	23

**Data Range**

In this paper, 441 samples of Middle East crude oil were used to correlate the bubble point pressure with gas specific gravity, oil specific gravity, solution gas-oil ratio

and reservoir temperature. The ranges of these parameters, which is used in this work, are shown in table 2.

**Table 2 Data range**

PVT parameter	Max	Min	Mean
$p_b$ , psia	7142	130	1623.195
API	55.37269	9.5	30.81534
$\gamma_g$	1.356	0.52	0.813692
$\gamma_o$	1.003546	0.7572	0.874407
$R_s$ , SCF/STB	1668	11	395.9524
$T$ , R	892	530	624.483

**Proposed correlation**

A new correlation was developed using the multiple nonlinear regression analysis and can be expressed as:

$$p_b = a_1 \gamma_g^{a_2} \gamma_o^{a_3} R_s^{a_4} T^{a_5} \exp(a_6 \gamma_g T)$$

With the coefficients  $a_0$  through  $a_7$  having the following values:

$$a_1 = 0.000221989 \quad a_2 = 0.07921662$$

$$a_3 = 2.882096242$$

$$a_4 = 0.787046015 \quad a_5 = 1.968344769$$

$$a_6 = -0.002128156$$

**Comparison of Correlations**

Average relative error, average absolute relative error, standard deviation, and correlation coefficient were computed for each correlation as shown in Table 3.

The bubble-point pressure correlation which was developed in this paper achieved the highest correlation coefficient of 0.98 with minimum average relative error of -0.56 %, minimum average absolute relative error of 8.12 %, minimum standard deviation of 10.69 %.

**Table 3 Statistical accuracy of bubble-point pressure correlations**

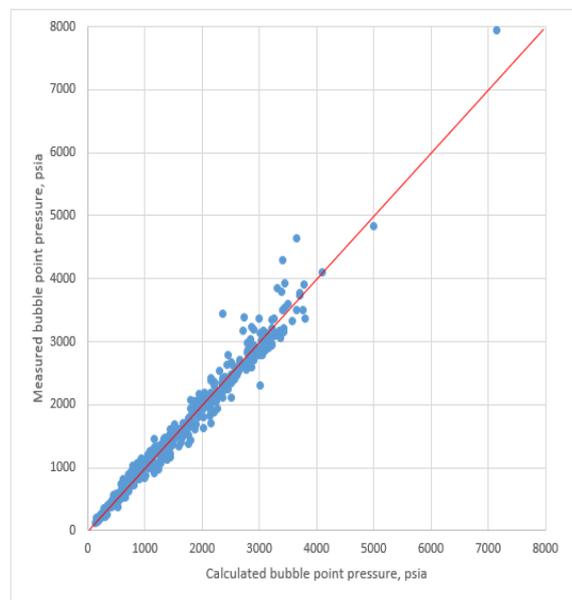
	ARE, %	AARE, %	SD	R2
Standing	-0.57	11.05	14.30	0.95

Lasater	6.53	10.97	14.91	0.97
Vazquez	-8.80	12.99	16.91	0.92
Glaso	-15.00	23.37	32.29	0.89
Al-Marhoun	-16.21	19.30	27.61	0.88
Dokla	-1.96	15.23	19.06	0.88
Macary	-39.31	53.34	62.54	0.72
Omar	-185.10	185.10	194.43	0.29
Petrosky	50.01	66.05	144.41	0.89
Kartaotmodjo	94.82	95.08	95.24	-0.28
Ghetto	-290.95	290.95	297.73	0.19
Farshad 96	-90.14	90.24	103.66	0.50
Almehaideb	-118.63	119.79	199.12	0.22
Hanafy	10.52	21.93	26.68	0.82
Khairy	-226.72	226.72	280.18	0.24
Al-Shammasi	-3.04	11.94	15.72	0.96
Velardi	-83.65	83.65	88.82	0.54
Dindoruk	-71.45	83.52	102.68	0.39
McCain	-52.36	52.36	57.73	0.65
Mehran	-71.97	71.97	75.28	0.57
Hemmati	-352.48	352.48	371.19	0.14
Ikiensikimama	-653.29	653.29	848.11	0.03
Moradi	-690.44	690.44	725.47	0.08
This study	-0.56	8.12	10.69	0.98

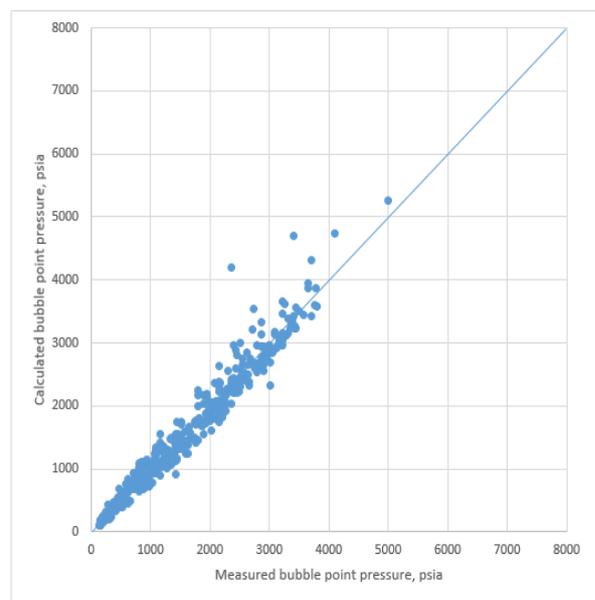
**Crossplot**

The crossplot of calculated values of bubble point pressure from this study’s correlation vs. measured values is presented on Figure 1. The plotted points of this study’s correlation fall very close to the perfect correlation of the 45° line.

The crossplots for the 23 other correlations were presented on the figures from 2 to 24.



**Figure 1. Crossplot of bubble point pressure (after this study)**



**Figure 2. Crossplot of bubble point pressure (Standing correlation)**

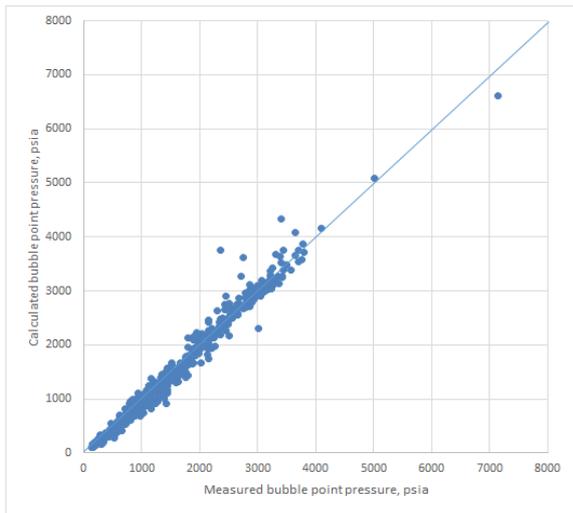


Figure 3. Crossplot of bubble point pressure (Lasater correlation)

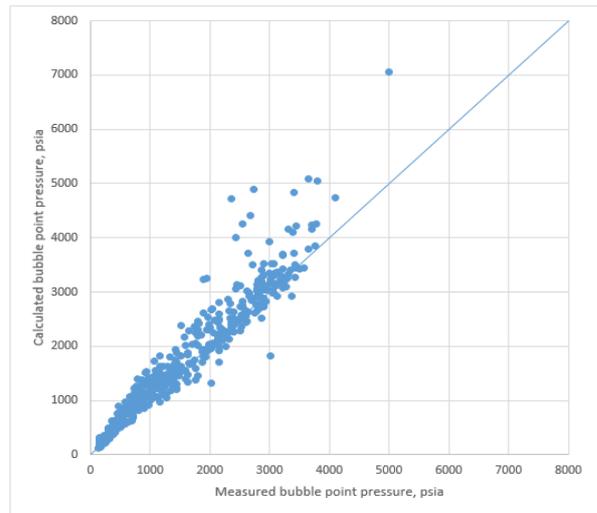


Figure 6. Crossplot of bubble point pressure (Al-Marhoun correlation)

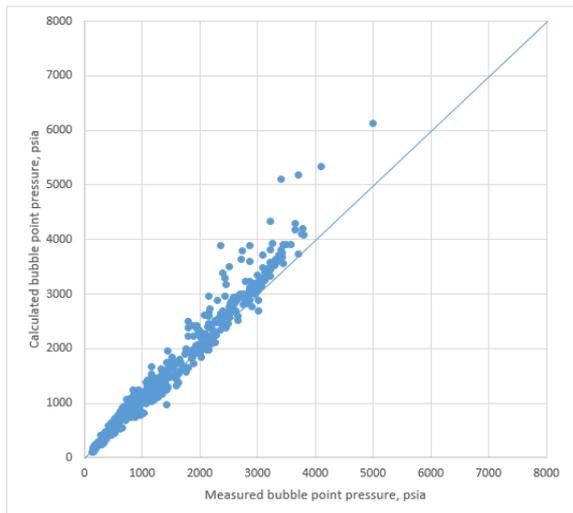


Figure 4. Crossplot of bubble point pressure (Vasques correlation)

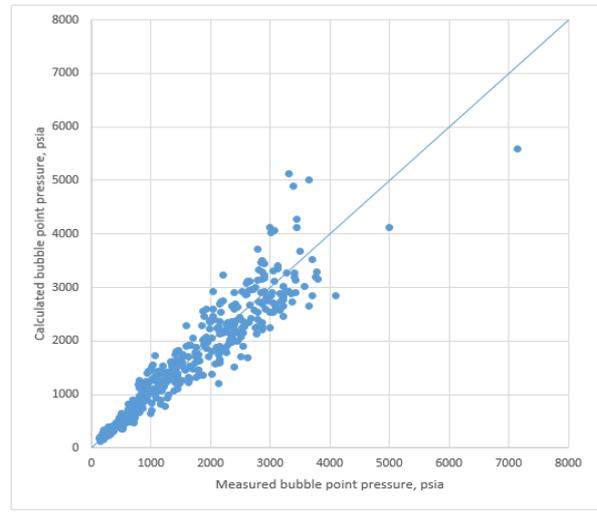


Figure 7. Crossplot of bubble point pressure (Dokla correlation)

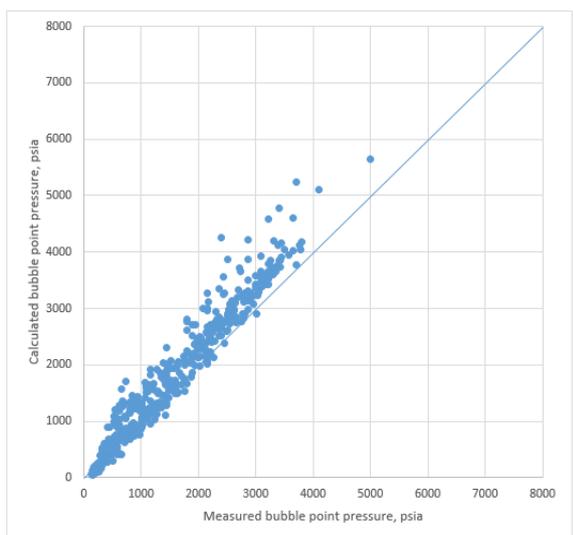


Figure 5. Crossplot of bubble point pressure (Glaso correlation)

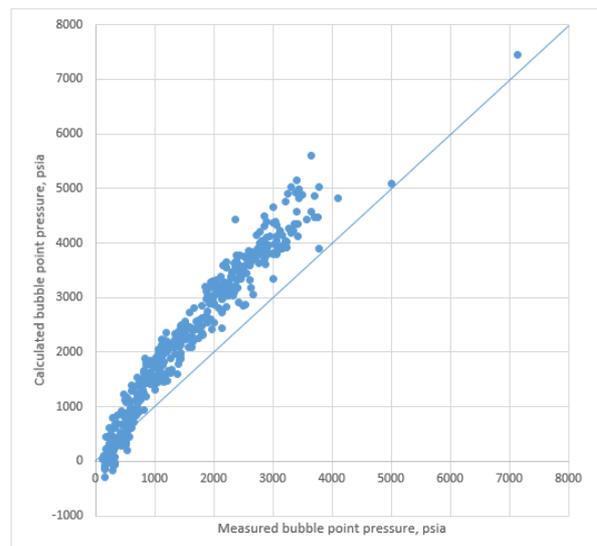
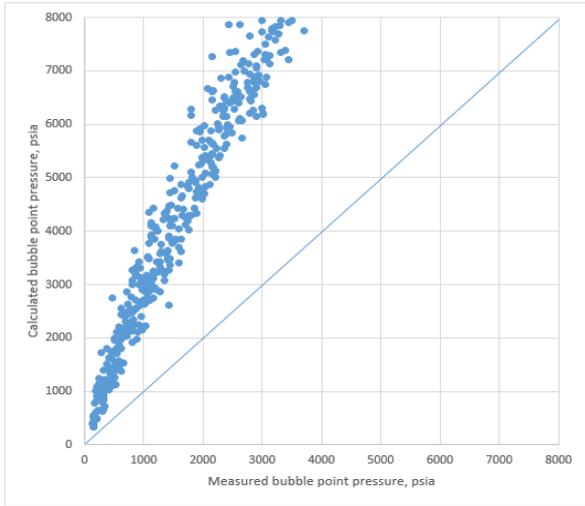
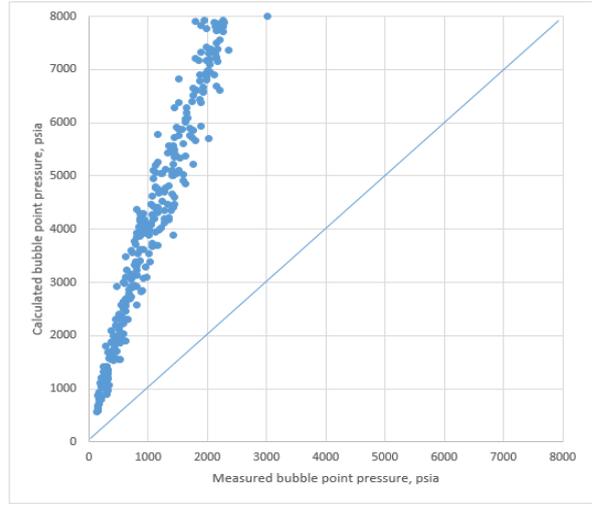


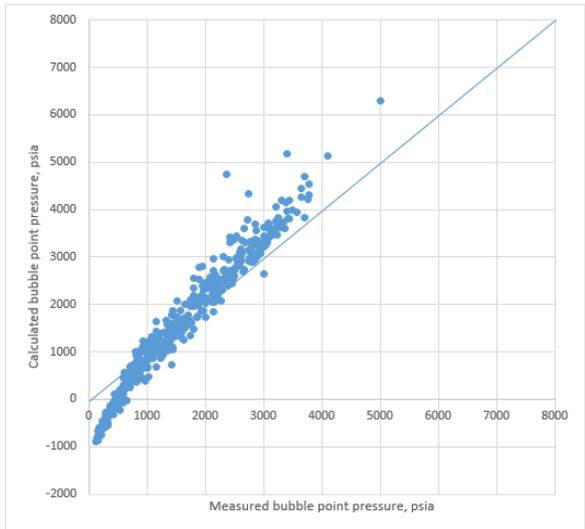
Figure 8. Crossplot of bubble point pressure (Macary correlation)



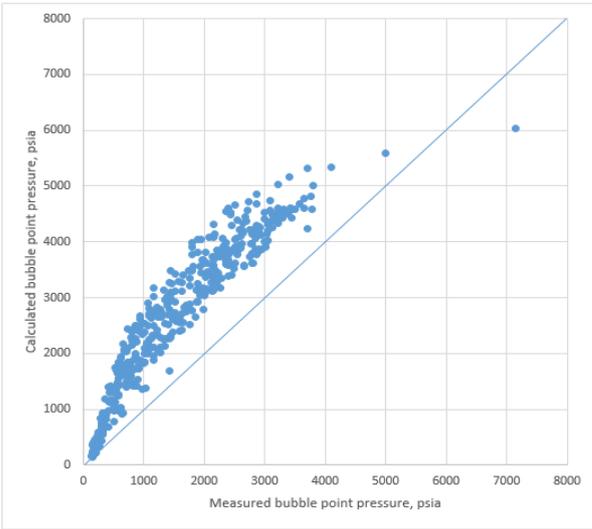
**Figure 9. Crossplot of bubble point pressure (Omar correlation)**



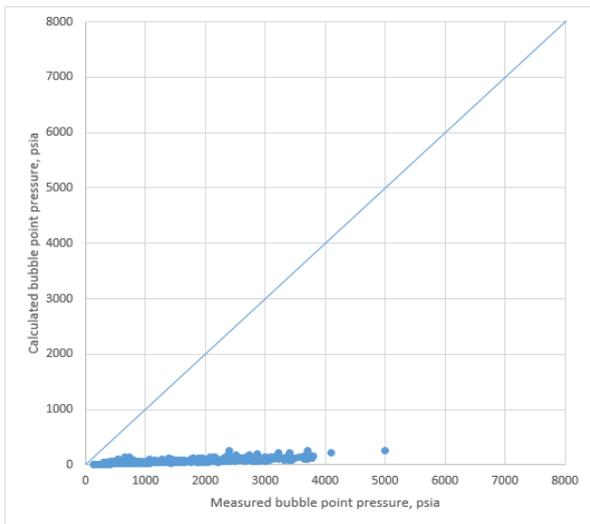
**Figure 12. Crossplot of bubble point pressure (Ghetto correlation)**



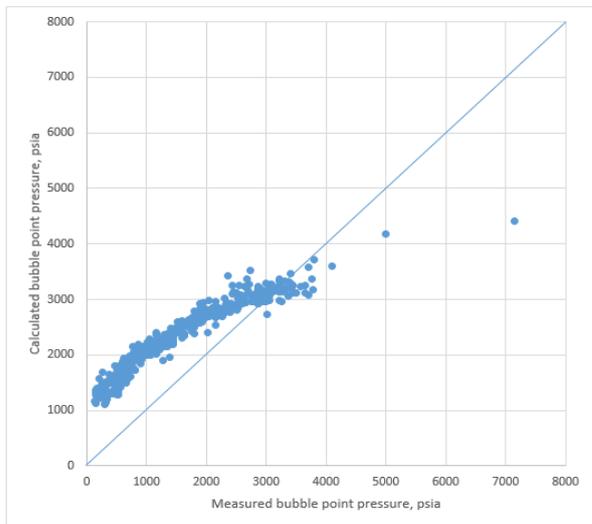
**Figure 10. Crossplot of bubble point pressure (Petrosky correlation)**



**Figure 13. Crossplot of bubble point pressure (Farshad correlation)**



**Figure 11. Crossplot of bubble point pressure (Kartoatmodjo correlation)**



**Figure 14. Crossplot of bubble point pressure (Almehaideb correlation)**

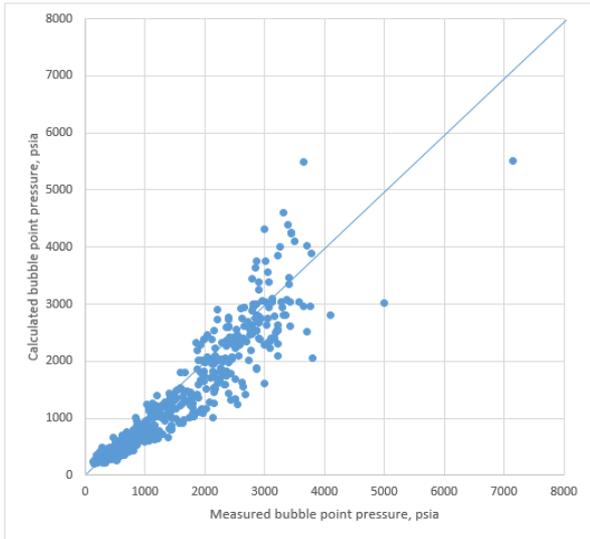


Figure 15. Crossplot of bubble point pressure (Hanafy correlation)

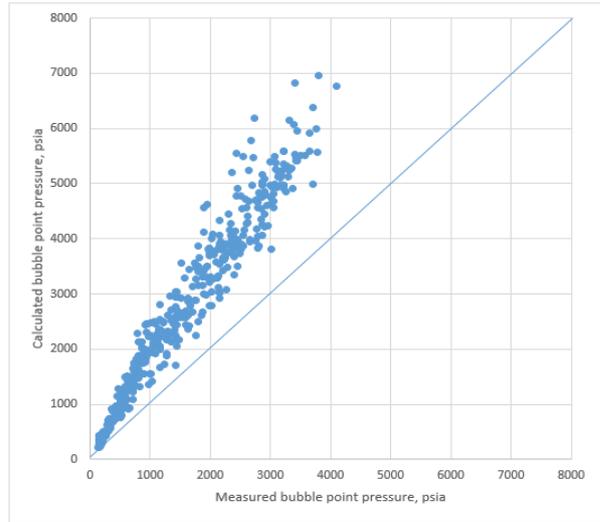


Figure 18. Crossplot of bubble point pressure (Velarde correlation)

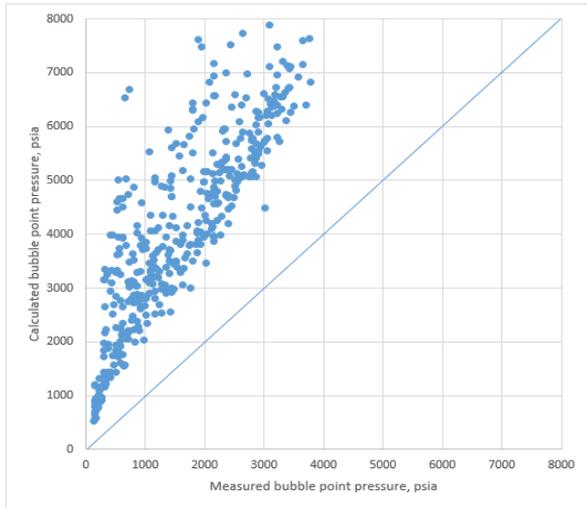


Figure 16. Crossplot of bubble point pressure (Khairy correlation)

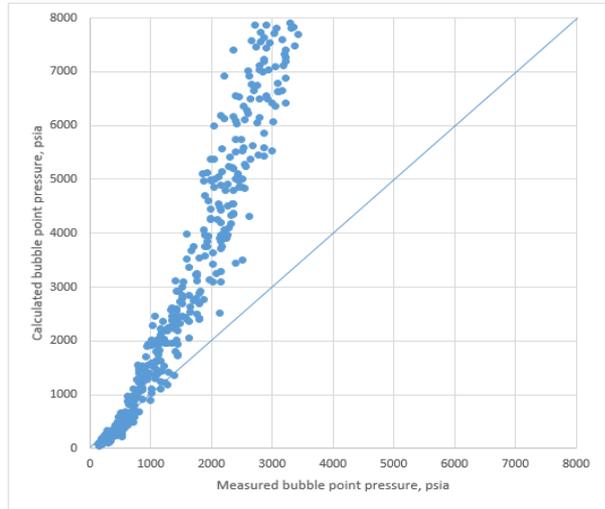


Figure 19. Crossplot of bubble point pressure (Dindoruk correlation)

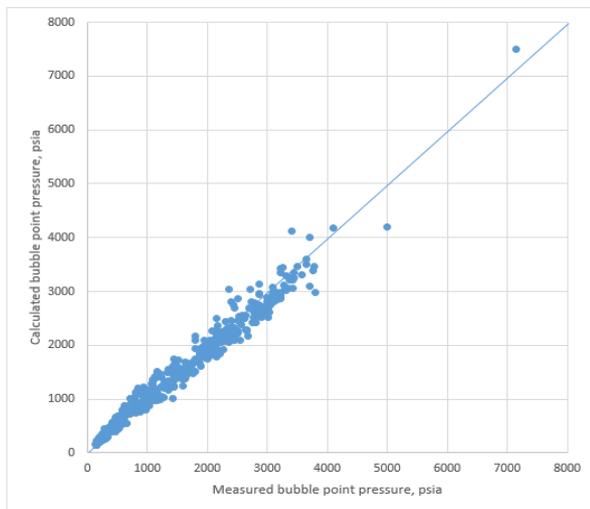


Figure 17. Crossplot of bubble point pressure (El-Shammasi correlation)

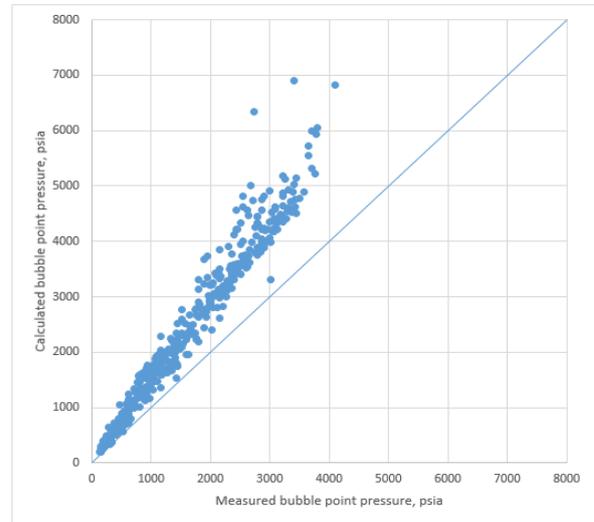


Figure 20. Crossplot of bubble point pressure (McCain correlation)

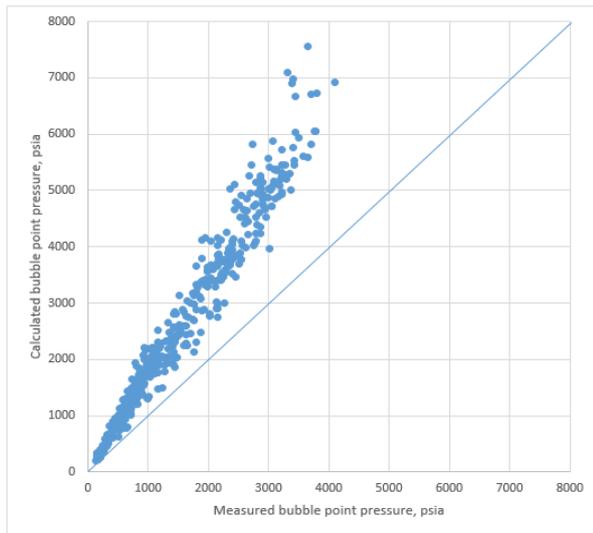


Figure 21. Crossplot of bubble point pressure (Mehran correlation)

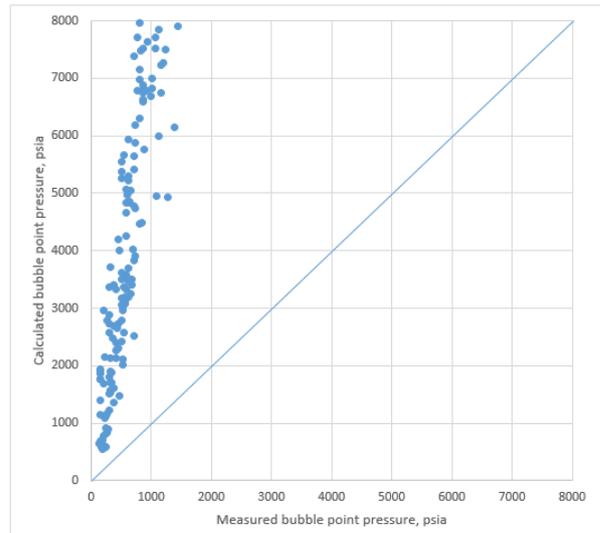


Figure 24. Crossplot of bubble point pressure (Moradi correlation)

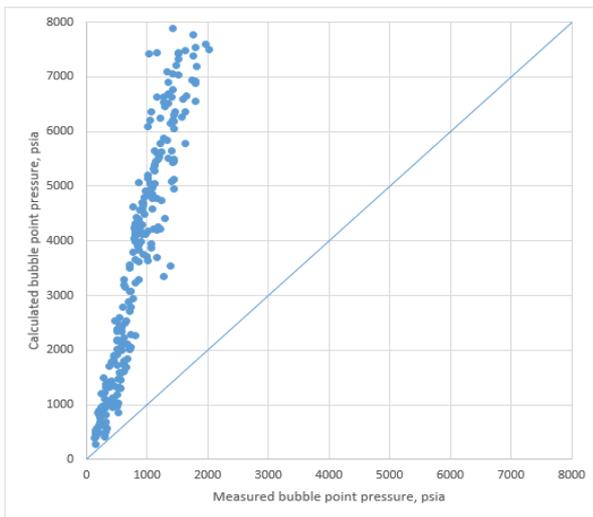


Figure 22. Crossplot of bubble point pressure (Hemmati correlation)

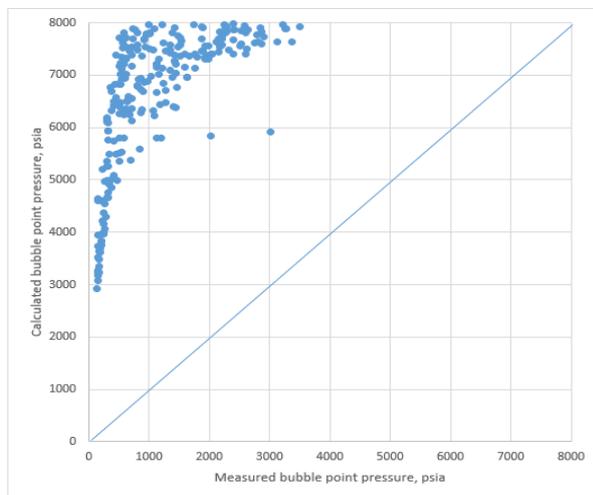


Figure 23. Crossplot of bubble point pressure (Ikiensikimama correlation)

### CONCLUSIONS

From this paper, one may conclude that:

1. This paper presents a comparison among 23 different correlations used to calculate the bubble point pressure of Middle East crude oil.
2. New bubble point pressure correlation was developed for Middle East crude oil.
3. Deviations from measured values of bubble point pressure indicated as average percent relative errors, average absolute percent relative errors, and the standard deviations, were lower for this study than for calculated values based on 23 different correlations.
4. The correlation coefficient of the correlations of this study are closer to one than that of other correlations.

### Nomenclature

- ARE = average relative error  
AARE = average absolute relative error  
 $T$  = reservoir temperature, °R  
 $p_b$  = bubble point pressure, psia  
 $R_s$  = solution gas-oil ratio  
 $\gamma_g$  = gas specific gravity  
 $\gamma_o$  = oil specific gravity

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The lower the value of  $E_i$  the more equally distributed are the errors between positive and negative values.

### Average Absolute Relative Error

This is defined as:

$$AARE = \sum_{i=1}^n |E_i| / n$$

and indicates the relative absolute deviation in percent from the tabulated values. A lower value implies a better correlation.

### Standard Deviation

Standard deviation  $s_x$  is a measure of dispersion and is expressed as:

$$s^2_x = \left( \sum_{i=1}^n E_i^2 \right) / (n - 1)$$

A lower value of standard deviation means a smaller degree of scatter.

### Correlation Coefficient

The correlation coefficient,  $r$ , represents the degree of success in reducing the standard deviation by regression analysis. It is defined as:

$$r^2 = 1 - \left[ \sum_{i=1}^n (p_{c\text{ cal}} - p_{c\text{ exp}})^2 / \sum_{i=1}^n (p_{c\text{ cal}} - p_{c\text{ avg}})^2 \right]$$

where

$$p_{b\text{ avg}} = \left( \sum_{i=1}^n p_{b\text{ exp}} \right) / n$$

The correlation coefficient lies between 0 and 1. A value of 1 indicates a perfect correlation, whereas a value of 0 implies no correlation at all among the given independent variables.

## Appendix A:

### Average Relative Error

This is an indication of the relative deviation in percent from the experimental values and is given by:

$$ARE = \left( \sum_{i=1}^n E_i \right) / n$$

$E_i$  is the relative deviation in percent of an estimated value from an experimental value and is defined by:

$$E_i = \left[ \frac{(p_{b\text{ exp}} - p_{b\text{ cal}})}{p_{b\text{ exp}}} \right] \times 100$$